Name: **KEY**
BSAD 210—Montgomery College

# EXAM 2

**Practice #2**

- There are 110 possible points on this exam. The test is out of 100.
- You have one class period to complete this exam, but you should be able to complete it in less than that
- Please turn off all cell phones and other electronic equipment.
- Be sure to read all instructions and questions carefully.
- Remember to show all your work. You may print your formulas in Excel using the Show Formulas option in the Formulas tab. Printed versions of your work showing formulas ***and*** showing the results counts as showing your work. But you must include both with your test for "showing your work" to count this way. Write your name on both print outs.
- Try all questions! You get zero points for questions that are not attempted.
- Note the last sheet lists all the equations you will need for this exam.
- *Please print clearly and neatly.*

**Part I: Matching.** *Write the letter from the column on the right which best matches each word or phrase in the column on the left. You will not use all the options on the right and you cannot use the same option more than once.*

2 points each.

1. **B** Alternative hypothesis
2. **I** Margin of error
3. **C** Null hypothesis
4. **E** Point estimate
5. **D** Proportion
6. **A** t distribution
7. **H** z distribution

A. Changes as n changes
B. Example: This person is allergic to cats.
C. Example: This person is not allergic to cats.
D. Example: Share of people who like cats.
E. Middle point for a confidence interval
F. Used when you know the population range
G. Used when you know the sample range
H. Used when you know the population standard deviation
I. What determines the size of a confidence interval

1. *The alternative hypothesis is the unusual thing; we assume people aren't allergic to cats thus the alternative hypothesis is that they are allergic. This doesn't mean a cat allergy is uncommon, but it's not something a clear majority of people have. In contrast, most people can see so being blind would be the alternative hypothesis.*
2. *The confidence interval is centered on the point estimate and the distance between either ends of the confidence interval and the point estimate is the margin of error. Thus, the margin of error is always half the confidence interval.*
3. *Since we assume any random person isn't allergic, that would be the null hypothesis. Even if many people are allergic, it would have to be a clear majority for it to reasonably be our default assumption.*
4. *When making a confidence interval, you add or subtract the margin of error from this point. That means that your point estimate—your best guess—is the middle of your interval.*
5. *A proportion is always bounded between zero and one. It's a percent (save growth rates which can exceed 100%) or a fraction or a chance. (To determine the chance or fraction, you take one number divided by another number. If this is the sample, the number in the denominator would be your n for any analysis because that's the number you're basing the sample on.) A share of people can't be more than 100%; this is a proportion.*
6. *The t distribution is really a family of distributions; which one you care about depends on the degrees of freedom, or n-1.*

7. *If you know sigma (σ), then you can use z scores. Knowing the population standard deviation means you don't have to rely on a family of distributions. You can rely on the standard normal distribution.*

**Part II: Multiple Choice.** *Circle the best answer to the following.*
4 points each.

8. A smaller α always results in:
    a. A larger critical z score
    b. A larger confidence level
    c. A smaller confidence level
    **d. A & B**
    e. None of the above

*Since alpha equals 100% minus confidence level, decreasing one must increase the other. A smaller alpha also means that the critical z score must increase. If there's a smaller chance to miss the true population parameter (for a confidence level), there the critical z score must be larger so it's more likely to include the parameter.*

9. Our definitions of type I error and type II error rely on understanding what the null hypothesis is. But it would be nice to have an accurate definition that does not rely on that technical detail. Which of the following definitions that don't refer to the null hypothesis describe type II error?
    a. Determining that an unusual thing is actually due to randomness.
    b. Mistakenly doing the unusual thing.
    **c. Mistakenly doing the usual thing.**
    d. Calculating a point estimate that's outside the confidence interval.
    e. None of the above makes sense

*"The unusual thing" cannot be the null hypothesis; the null hypothesis is the default position for the typical individual, such as not hiring someone or assuming they are innocent. The unusual thing is rejecting the null hypothesis and the usual thing is failing to reject the null hypothesis. If you mistakenly did the usual thing, you failed to reject the null but should have rejected it.*

10. We say "fail to reject" the null hypothesis rather than "accept" the null hypothesis. Why?
    a. Because we don't have evidence that the null is correct.
    b. Because the difference between the parameter and the statistics is not zero.
    c. Because there is always randomness.
    **d. A & B**
    e. None of the above

*There is not enough evidence to suggest the null is false. It could be false, of course, but we don't really know. Hypothesis testing is set up to determine if there's enough evidence to reject the null, not to determine which hypothesis is true.*

*Keep in mind the chance that the gathered statistic (x-bar) is exactly the same as the point of reference (μ) is infinitesimally small. In practice, the numerator of the hypothesis testing will always be something other than zero. If we fail to reject the null hypothesis, we can't say the null is "true." We didn't find "no" difference, we did not find a statistically significant difference.*

*Think of a trial. A person is not found "innocent;" she is found "not guilty." We're not saying the person is innocent—"proving" innocence is functionally impossible—so we say "not guilty."*

11. A machine should puts eight pounds of flour in a bag. Miguel tests if the machine is working well by weighing six randomly selected bags. The average weight of these bags is 6.1 pounds with a sample standard deviation of 1.2 pounds. What should Miguel conclude?
    a. It's statistically significant at the 90% level.
    **b. It's statistically significant at the 95% level and lower.**
    c. It's statistically significant at the 99% level and lower.
    d. It's statistically significant at the 99.9% level and lower.
    e. It's not statistically significant.

*First, note this is a two-tailed test. It's also a t-test as we don't know the population standard deviation. You should get about 3.878* $(=|(6.1-8.0)/(1.2/\sqrt{6})|)$ *for the calculated value. The critical t score for 99% significance (5 degrees of freedom) is 4.032. For 95%, it's 2.571. Thus, it's statistically significant at the 95% level but not the 99% level.*

12. Suppose a flight time follows a normal distribution with an average of 2.5 hours and a standard deviation of ten minutes. What's the probability that the flight will last longer than 2 hours and 45 minutes (2.75 hours)?
    a. 0.0130
    b. 0.4900
    c. 0.9332
    d. It is impossible to tell with the information provided.
    **e. It is possible to tell but the option is not listed here.**

*Note all units have to agree: 2.5 hours becomes 150 minutes, for example. Using =NORM.DIST(165,150,10,1) will tell you the chance that it will last 2.75 hours (2* *hours and 45 minutes) or less time (0.9332). But we want <u>longer</u> than 2.75 hours. 1- 0.9332 = 0.0668 is the answer.*

13. William Wails wonders which whistle is best for his whale-watching work. His whistle is important. When he sees a whale on a whale-watching trip, he can use the whistle to attract the attention of his passengers, but it's hard to hear over the wind and seas. His current whistle attracts 80% of passengers. He tries a new whistle and it attracts 83% of passengers. If you were to run an analysis on if this whistle is better, you need more information. What information do you need?
    a. How many trips he took.
    b. The cost of the new whistle.
    c. The standard deviation.
    d. A & C
    **e. What you need isn't listed here.**

*You need the number of passengers; that's your n, not the number of trips. Because this is a test for proportion, the standard deviation is determined by $\pi$ (see the equation).*

*Interestingly, the best answer besides E is B. A and C are completely irrelevant but if you also wanted to know if this whistle was a <u>practical</u> improvement, you'd need to know how much the whistle cost. If it cost, say, $40,000 (that's some whistle!), then an additional three percentage points wouldn't mean much.*

14. Which idea explains why we can never "prove" anything in statistics?
    **a. The central limit theorem**
    b. Hypothesis testing
    c. The types of error
    d. The confidence level
    e. None of the above

*The CLT argues that sample means follow a normal distribution. Any sample mean we have may be a usual sample mean but it may be, due to random variation, unusually high or low. "Prove" is a word that comes with a level of certainty we cannot claim. That's why alpha—the chance that there's nothing unusual going on—is never zero. There's always a chance (however small) that we got a sample mean that was just odd.*

15. What's the difference between practical significance and statistical significance?
    a. Statistical significance only occurs if there's practical significance.
    b. Practical significance focuses on z scores while statistical significance uses t scores.
    **c. Statistical significance is the math side of the analysis while practical significance is more the business side of the analysis.**

d. A & C
e. None; they are functionally the same thing.

*Statistical significance relates to rejecting or failing to reject the null hypothesis; practical significance concerns if a difference is notable. Note that if it's not statistically significant, it can't be practical significant because it's not even clear there's a difference in the first place so it can't possibly be notable.*

16. Muhammad is running for a local election. He surveys 82 people and finds that 46 will vote for him. At 99.9% confidence, determine Muhammad's confidence interval.
    - **a. Between 0.381 & 0.741**
    - b. Between 0.392 & 0.730
    - c. Between 0.420 & 0.702
    - d. Between 0.433 & 0.689
    - e. None of the above

*This is a proportion confidence interval so we should use the equation found in the equation and information section. We also note that Muhammad's sample proportion is about 0.561 and, at 99.9% confidence, z=3.291.*

$$\widehat{CI}_{\bar{p}} = 0.561 \mp 3.291\sqrt{\frac{0.561(1-0.561)}{82}} = 0.561 \mp 0.18$$

*That's a pretty big range! This isn't entirely surprising given the confidence level but even at 95% confidence, the range is between 0.454 and 0.668. Muhammad should increase his sample size.*

17. Referring to the previous question, what's the smallest number of people Muhammad would have to survey to get a margin of error of no more than 0.03 (3%)?
    - a. 1,844
    - b. 2,653
    - c. 2,964
    - **d. 3,009**
    - e. None of the above

*To find the optimal sample size, use this equation:*

$$n = \left(\frac{z_{\frac{\alpha}{2}}}{ME}\right)^2 \bar{p}(1-\bar{p})$$

*But there's a problem with this equation: we don't know* $p$*-bar. As mentioned, if you're not sure what to use—you don't have previous work to draw on, for example, then set p-bar equal to 0.5 (because that maximizes the standard deviation). So:*

$$n = \left(\frac{3.291}{0.03}\right)^2 0.5(1-0.5) = (12{,}034.09)(0.25) = 3{,}008.523, \mathit{or}\ 3{,}009$$

18. Aziz wants a guarantee for his shipping company, reimbursing customers for unusually slow deliveries. Suppose the time it takes for a shipment to arrive follows a normal distribution with a mean of 20.6 days and a standard deviation of 3.1 days. If Aziz wants to reimburse no more than 1% of customers, how many days should he set his guarantee at?
    a. 13
    b. 14
    c. 27
    **d. 28**
    e. None of the above

*The first thing to remember is that bigger numbers, not smaller numbers, means slower deliveries. If you selected either (A) or (B), recognize you want to reimburse customers that got their stuff in* **less** *time than average. Clearly, the deadline has to be larger than 20.6. Use 0.99, not 0.01.*

*Using NORM.INV(0.99,20.6,3.1) results in 27.812. Note Aziz should round up. If he rounded down, and set the threshold at 27 days, Aziz would reimburse more than 1% of his customers.*

19. Which of the following is/are true?
    a. "The CONFIDENCE function in Excel outputs the confidence interval."
    b. "A confidence interval made with a critical t score will be smaller than a confidence interval made with a critical z score (holding all else constant)."
    c. "A larger confidence interval is always preferred to a smaller one because you're more likely to include the population mean."
    d. B & C
    **e. None of these are true**

*I was tempted to not put A as an option because the answer is in the reference section at the end of the test. But I've seen many students, even with that information, think that the CONFIDENCE function gives them the full interval. But recall it only gives you the margin of error; the actual interval itself is found by adding and subtracting that margin of error from the mean.*

*Critical z scores are always smaller than t score (which increase as degrees of freedom fall) because t scores assume less information. Thus B is false: the confidence interval made with critical t scores will be larger because the margin of error will be larger.*

*While the second clause of C is true—a larger interval is more likely to include the population mean—it's not true that larger intervals are always more valuable. Larger intervals come with a big problem: vagueness. Saying, for example, that a politician will get between 0% and 100% of the vote is true but too vague to be helpful. Smaller intervals means more useable information at the cost that they may miss the mark entirely.*

**Part III: Short Answer.** *Answer the following.*
16 points each.

20. Zachary Zambango is analyzing the amount of traffic at an intersection as part of a larger project to understand traffic patterns. He needs to know about how many cars pass through the intersection each weekday. After sampling 10 days, the sample average is 417.8 cars with a sample standard deviation of 38.8 cars. Give the confidence interval for 95% confidence and 99% confidence.

*First, note that this will use the t distribution because we don't know the population standard deviation; we must rely on the sample standard deviation.*

*This is simply a matter of using the CONFIDENCE.T function to get the margin of error. You'll need to do it twice, one for each confidence level. Note that the function requires alpha so you have to convert the confidence levels into alphas: 95% becomes 0.05 and 99% becomes 0.01.*

*CONFIDENCE.T(0.05,38.8,10) = 27.76*
*CONFIDENCE.T(0.01,38.8,10) = 39.87*

*Now we add and subtract each of these numbers from the sample average to get two ranges.*

*95%: 390.04 to 445.56*
*99%: 377.93 to 457.67*

21. Suppose Octavian College advertises that 80% of its graduates find a job that requires a college degree within one year of getting their degrees. You wonder if this is accurate and

test it by talking to 106 randomly selected recent graduates. Of those 100 recent graduates, 77 of them found a job that required a college degree. Is Octavian College's claim accurate?

In answering this question, be sure to:

- Show your work on the calculated value;
- Indicate what your calculated value is;
- Indicate what your critical values are; and
- Determine if this is statistically significant

*This is a test of proportion, with the sample proportion, p, equaling (77/106) 0.7264.*

$$z_p = \left| \frac{0.7264 - 0.8}{\sqrt{\frac{0.8(1-0.8)}{106}}} \right| = \left| \frac{-0.0736}{\sqrt{\frac{0.16}{106}}} \right| = \left| \frac{-0.0736}{\sqrt{0.0015}} \right| = \left| \frac{-0.0736}{0.0389} \right| = 1.8920$$

*Since this is a two-tailed z score, the relevant critical values are 1.645 (90%) and 1.960 (95%). 90% is a sad level to have to claim and I largely include it for illustrative purposes. In practice, I wouldn't call this statistically significant because 1.892 < 1.96.*

22. I showed you how to derive the Empirical Rule in Excel. Using that same technique, determine a "new" empirical rule for 0.5, 1.5, and 2.5 standard deviations from the mean.

*We can more or less follow the instructions given when we did this at the beginning of the unit but I encourage you to try to figure it out on your own without looking at the notes. As with everything else, you learn more when you try to figure it out rather than just copying what you saw.*

*First, I set up the boundaries, as defined above, going in each direction.*

| | | | | | |
|---|---|---|---|---|---|
| -2.5 | -1.5 | -0.5 | 0.5 | 1.5 | 2.5 |

*Then I used the NORM.S.DIST function to find the area under the curve for each and got this:*

| | | | | | |
|---|---|---|---|---|---|
| -2.5 | -1.5 | -0.5 | 0.5 | 1.5 | 2.5 |
| | | | | | |
| 0.00621 | 0.066807 | 0.308538 | 0.691462 | 0.933193 | 0.99379 |

*Then I subtracted the negative value from the positive value (e.g. NORM.S.DIST(0.5) – NORM.S.DIST(-0.5)) and got this:*

| | | | | | |
|---|---|---|---|---|---|
| -2.5 | -1.5 | -0.5 | 0.5 | 1.5 | 2.5 |
| | | | | | |
| 0.00621 | 0.066807 | 0.308538 | 0.691462 | 0.933193 | 0.99379 |
| | | | | | |
| | | 38.3% | | | |
| | 86.6% | | | | |
| 98.8% | | | | | |

*Here it is with the formulas (note I used an older version of Excel so my commands don't perfectly match).*

| | | | | | |
|---|---|---|---|---|---|
| -2.5 | -1.5 | -0.5 | 0.5 | 1.5 | 2.5 |
| | | | | | |
| =NORMSDIST(C35) | =NORMSDIST(D35) | =NORMSDIST(E35) | =NORMSDIST(F35) | =NORMSDIST(G35) | =NORMSDIST(H35) |
| | | | | | |
| | | =F37-E37 | | | |
| | =G37-D37 | | | | |
| =H37-C37 | | | | | |

## Exam 2 Equation and Information Reference

| *Function* | *Output* |
|---|---|
| ABS | The absolute value of an input |
| AVERAGE | Arithmetic mean of a dataset |
| CONFIDENCE.NORM | Determines the margin of error to make a confidence interval (known σ) |
| CONFIDENCE.T | Determines the margin of error to make a confidence interval (unknown σ) |
| CORREL | Correlation coefficient of two variables |
| CTRL + ` | Show formulas |
| CTRL + F | Find |
| CTRL + P | Print |
| CTRL + X | Cut highlighted area |
| CTRL + C | Copy highlighted area |
| CTRL + V | Paste highlighted area |
| CTRL + Z | Undo |
| F4 | Makes cell reference absolute |
| GEOMEAN | Geometric mean of a dataset (adjustments must be added manually) |
| LARGE | Larger values of a dataset (k=1 is largest, k=2 is second largest, k=3 is third largest…) |
| MAX | Maximum value of a dataset |
| MEDIAN | Median of a dataset |
| MIN | Minimum value of a dataset |

| | |
|---|---|
| MODE | Mode of a dataset |
| NORM.DIST | Returns the normal distribution for a specified mean and standard deviation. |
| NORM.INV | Returns the inverse of the normal cumulative distribution for a specified mean and standard deviation. |
| NORM.S.DIST | Returns the standard normal distribution. |
| NORM.S.INV | Returns the inverse of the standard normal cumulative distribution. Useful for finding critical z scores. |
| QUARTILE | The $0^{th}$ to $4^{th}$ quartile of a dataset |
| SQRT | Finds the square root of the value in question. |
| SMALL | Smaller values of a dataset (k=1 is smallest, k=2 is second smallest, k=3 is third smallest…) |
| STDEV.S | Standard deviation of a sample |
| T.INV | Finds area under a t distribution; useful for finding one-tailed critical t scores. |
| T.INV.2T | Finds area under a t distribution; useful for finding two-tailed critical t scores. |

*Coefficient of Variation*

$$CV_{sample} = \frac{s}{\bar{x}}(100)$$

*Confidence interval for proportion*

$$\widehat{CI}_{\bar{p}} = \bar{p} \mp z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

*Optimal Sample Size*

$$n = \left(\frac{z_{\alpha/2}\sigma}{ME}\right)^2$$

$$n = \left(\frac{z_{\alpha/2}}{ME}\right)^2 \bar{p}(1-\bar{p})$$

*Critical z scores*

Use =NORM.S.INV command

| *Confidence* | $\alpha$ | $z_{\alpha/2}$ | $z_{\alpha}$ |
|---|---|---|---|
| 90% | 0.1 | 1.645 | 1.280 |
| 95% | 0.05 | 1.960 | 1.645 |
| 99% | 0.01 | 2.576 | 2.330 |
| 99.9% | 0.001 | 3.291 | 3.090 |

*Critical t scores*

Use T.INV or T.INV.2T commands

*z-test*

$$z_{\bar{x}} = \left|\frac{\bar{x} - \mu_{H_0}}{\sigma/\sqrt{n}}\right|$$

*Proportion*

$$z_p = \left|\frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}\right|$$

*t-test*

$$t_{\bar{x}} = \left|\frac{\bar{x} - \mu_{H_0}}{s/\sqrt{n}}\right|$$