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LECTURE 23: AVOIDING TYPE II ERRORS II

I. Power

a. The CLT cuts both ways. Just as the hypothesized distribution is centered on your null hypothesis, there's an actual distribution centered on your sample mean.

   i. Extreme values on your hypothesized distribution reflect the chance for Type I error.

   ii. Extreme values on your actual distribution reflect the chance for Type II error.

b. Recall our earlier example: if we had a result of less than 102.8 miles, we would fail to reject our null hypothesis. What if, by chance, our sample was under that? Perhaps something happens to be strange about most of those cars and the sample mean should actually be much lower? What if we made a Type II error?

   i. Now we imagine our sample mean as the null hypothesis; it is our best estimate, after all.

   ii. We also imagine the 102.8 as the mean we could have gotten.

$$z = \frac{102.8 - 105}{12/\sqrt{50}} = \frac{-2.2}{12/\sqrt{50}} = \frac{-2.2}{1.697} = -1.30$$

   iii. Finding β involves looking up that value on the z-table:

1st digit	2nd digit									
z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379

   iv. At a z-score of -1.30, the value is 0.0968. That value is our β.

c. Important in this discussion is the hypothesis test's *power*, or the probability of it correctly rejecting the null hypothesis.

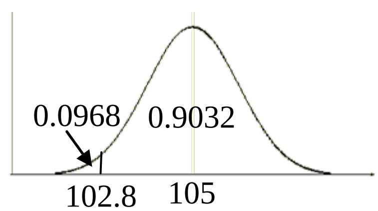
$$Power = 1 - \beta$$

i. In the example above, the power of the test is 0.9032; there is a 90.32% chance of correctly concluding the new batteries are effective.

II. Too High or Too Low?

a. It's not a simple matter of "the z-score is always the β." Sometimes it's the power; it depends on if, when determining the critical sample mean, you added or subtracted.

b. The z-score, by definition, results in the portion of a normal distribution below that value. In this example,



c. But suppose instead lower values, not higher values, indicated statistical significance. Suppose we weren't testing how far a battery would take us but how long it took to charge the battery.

i. Suppose it takes 12 hours to charge a battery with a standard deviation of 1.5 hours. Our new battery takes 10.89 hours based on a sample of 16 batteries.

ii. At 95% confidence, our new critical sample mean is:

$$\bar{x}_\alpha = 12 - 1.645\left(\frac{1.5}{\sqrt{16}}\right) = 12 - 1.645(0.375) = 11.38\ hours$$

iii. Note that since lower numbers are now interesting, we subtract.

iv. Here's the z-score:

$$z = \frac{11.38 - 10.89}{1.5/\sqrt{16}} = \frac{-0.5}{0.375} = 1.31$$

d. Note the value is now positive. That makes sense as we're curious about a different tail. So, let's look at the z-table.

1st digit	2nd digit									
z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545

e. Now here's the key: remember what the z-score represents. It's the portion of the values below the z-score in question. In this case, that includes our sample mean of 11. In other words, 0.9049 is not β anymore; it is the test's power. The β is now 0.0951.