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LECTURE 21: EXPECTED VALUE

I. Expected value

a. Suppose I roll a four sided die and pay you a dollar if the result is even and nothing if the result is odd. How much are you willing to pay to play this game?
   i. What if you only get paid if the result is a one?
   ii. Or suppose I pay you, in dollars, the result of the die? (Rolling a "2" gets you $2; a "4" gets you $4.)

b. What game would you rather play?
   i. If the die is even, you lose $1 and if odd, you get $1.
   ii. If the die is even, you lose $10 and if odd, you get $10.

c. Both questions can be aided by *expected value*, or the value of a random payoff. (Expected value is also called the mean, or average).
   i. To determine expected value, multiply the payoff of each scenario by the probability of that scenario happening. Then add these values together.
   ii. So for the game in I.a.ii., the expected value is:

$$\frac{1}{4}\$1 + \frac{1}{4}\$2 + \frac{1}{4}\$3 + \frac{1}{4}\$4 = \$2.50$$

II. Attitudes towards risk

a. We have not yet answered the question from the beginning: how much are you willing to play these games?

b. Just because we know the expected value, doesn't mean we know how much people are willing to pay. The very idea of a risky payoff can be exciting for some and nerve wrecking for others.

c. It's useful to compare expected value, E(x), with a certain amount of A, where A = E(x).
   i. *Risk loving* individuals prefer E(x) to A.
   ii. *Risk neutral* individuals are indifferent to E(x) and A.
   iii. *Risk averse* individuals prefer A to E(x).

d. Different people have different risk preferences and these preferences can vary for the same person from time to time. Some industries are built on how people approach risk.

e. The insurance industry is built around people being risk averse. People find more pain in facing –E(x) compared to –A, where –A is the insurance price and –E(x) is the expected cost of an accident. They would rather lose A than lose E(x).
    i. Suppose each person has a 1% chance of having an auto accident which causes $1,000 in damages, or an expected cost of $10. Suppose each person is willing to pay $15 to get insurance.
    ii. With, say, 50,000 customers, the insurance company receives $750,000 in revenue and pays out only $500,000 in damages, leaving room left over to pay for the costs of business and secure a healthy profit.

f. The gambling industry is built around people being risk loving. People prefer E(x) to A, where A is the cost to play a game and E(x) is the expected payoff from that game. They will give up A to get E(x).
    i. Roulette is a game where you spin a disc with 38 spaces and a spinning ball that eventually lands on one of those spaces. If it lands on a space of yours, you get 35 times what you paid in (one dollar becomes $35), and lose the dollar otherwise.
    ii. Since the probability of success is 1/38, the expected value of winning is $35/38, or $0.92. The probability of losing is 37/38 so the expected value of losing is -$0.97. Added together, every player of roulette loses, on average, five cents per dollar gambled; the casino gets five cents. If $100,000 are gambled on roulette, the casino walks away with $5,000 and the customers leave $5,000 poorer.

g. When it comes to income, most people are risk averse. They prefer a steady income than a fluctuating one. Remembering compensating differentials, jobs with a fluctuating income should pay higher, on average, than those with a steady income.

h. Note that a person's attitude towards risk is ***not*** the same as her attitude towards uncertainty.
    i. People tend to be either risk loving, risk neutral, or risk averse. We compare a riskless payoff to a probabilistic one to see.
    ii. But people tend to be averse to ambiguity. They prefer the probabilistic payoff to an unquantifiable payoff.

III. Audit Example

a. Let's revisit another example from the last set of notes: time to complete an audit.

b. We have a nice table indicating the various times it will take to complete an audit:

Total Time	Probability
4 weeks	0.08
5 weeks	0.16
6 weeks	0.34
7 weeks	0.42

c. But customers don't always want a range. Sometimes they want a clear, single number. We use what we've learned to find that number.
d. Simply multiply each probability by the appropriate completion time. Then add them together.
    i. Note this is a weighted average: the probabilities are the weights. And because the total probabilities add up to one, there is no need for an additional step of division.
e. So let's multiply and add:

Time (weeks)	Probability	Expected (weeks)
4	0.08	0.32
5	0.16	0.80
6	0.34	2.04
7	0.42	2.94
Expected Time (weeks)		**6.10**

f. A better estimation is a little more than 6 weeks.

IV. Government Contract Example
a. Uriel wants to get a government contract for his small company. Suppose the application process costs Uriel $50,000. Also suppose that there's a 30 percent chance of getting the contract.
    i. If he gets the contract, there's a 10 percent chance of doing the job quickly and 90 percent chance of doing the job slowly.
    ii. Doing the job quickly will earn a total profit (after all costs) of $300,000; doing the job slowly will earn a total profit (after all costs) of $200,000.
b. What is the expected value of pursuing the contract?
    i. To do this question, understand there are three possibilities:
        1. Getting the contract and doing the job quickly.
        2. Getting the contract and doing the job slowly.
        3. Not getting the contract at all.
    ii. We know the probabilities of (1) and (2): 0.3*0.1 and 0.3*0.9; but what's the probability of not getting the contract at all?

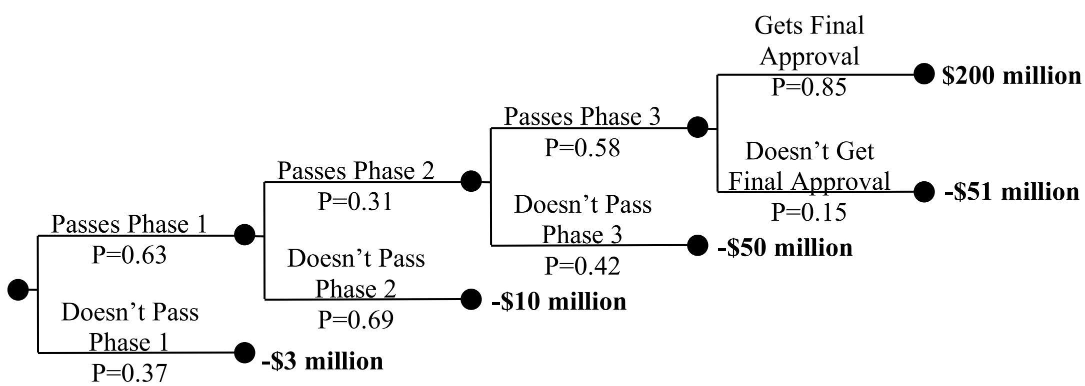
1. That's 0.7 (if there's a 30 percent chance of getting the contract, then there's a 70 percent chance of not getting the contract).

iii. Now we multiply the chance of each scenario by the associated payoff:
   1. 0.3*0.1*$300,000 = $9,000
   2. 0.3*0.9*$200,000 = $54,000
   3. 0.7*(-$50,000) = -$35,000

iv. Adding them together and we get $28,000. Is it a good idea to try to get this contract? It depending on what else Uriel could be doing with the time and money spent to try to get it, but now we have a number to work with.

V. Drug Example

a. Let's apply the drug example from before by assigning profit results at the end of each outcome. Note that most of these will be negative: if the drug is not approved then the company paid a lot for trials but got no revenue. These profit numbers are in millions of dollars and are fictional.



b. Math time!

i. 0.37*(-$3) + 0.63*0.69*(-$10) + 0.63*0.31*0.42*(-$50) + 0.63*0.31*0.58*0.15*(-$51) + 0.63*0.31*0.58*0.85*($200) = $8.83

ii. We expect this drug will bring in almost $9 million in profit. Note how much lower than that is compared to the $200 million in profit we'll get if it's approved. But, as they say, that's a big if.