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LECTURE 10: CONFIDENCE INTERVALS II

I. Calculating the Margin of Error with Unknown σ

a. We often don't know σ. This requires us to rely on the sample's standard deviation, *s*. This changes everything.

b. Recall that *s* is influenced by sample size, *n*. Bigger sample sizes means we can more reasonably estimate the standard deviation. When *n* gets really large, there's no practical difference between knowing and not knowing σ.

c. But most the time, the sample is good but not enormous. Thus we cannot use our perfect normal distribution. We have to use a different distribution: Student's t-distribution.

   i. Like a normal distribution, the t-distribution is bell-shaped and symmetric around the mean.

   ii. The t-distribution is flatter and wider than the normal distribution.

   iii. The t-distribution depends on the *degrees of freedom*, or the number of values that are free to vary given that certain information is known.

      1. One bit of information that's known for any sample is the sample mean. Therefore, the degrees of freedom (df) for our use here is equal to n – 1.

   iv. The t-distribution is a family of distributions which change based on the degrees of freedom.

d. Here's the equation:

$$\widehat{CI}_{\bar{x}} = \bar{x} \mp t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

   i. The hat over CI reminds us that this is an approximation. Note how similar this equation is to the previous one.

e. Below is a table for the t-distribution with single tail alpha on top and two tail confidence levels on the bottom. Note that as df increases, the critical values approach the z-score.

Table B *t* distribution critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

II. Critical T-values

a. Excel has a CONFIDENCE.T function for calculating the interval with a t distribution. It also has the whole table for t-values.

b. The command for this is “=T.INV.2T”

    i. *Probability* is the alpha value, assuming two tails.

    ii. *Deg_freedom* is the degrees of freedom: n – 1.

c. So if you want to know the critical t-value for an alpha of 0.05 with six degrees of freedom, you’d type “=T.INV.2T(0.05,6)” and press ENTER.

i. You should get about 2.447, the same as in the table under 95% confidence with 6 degrees of freedom.
ii. Note there is a version without the 2T; the 2T stands for two tails. Without it, you're assuming alpha is only in one tail.
iii. If you think the table claims the result should be 1.943 (or -1.943), it's because you're looking at the one-tailed version and not the two-tailed version. (In the two-tailed version, that 0.05 is equally split between two sides of the distribution, thus a single tail would have a value of 0.25.)

III. Additional Notes on Confidence Intervals

a. Choosing a confidence level is tricky, which is why sometimes you see multiple levels reported. That's not always the case, though, because—bizarrely—people often want definite answers when it comes to statistics.
   i. On one hand, a narrow range tells you a lot about what the population mean might be. You are more precise but risk completely missing the mark.
   ii. On the other hand, a wider range ensures the population mean is in that band. Your range probably includes the parameter, but you're vague.
   iii. The question is, what side is best to err on? That changes with circumstance. That said, 90% confidence is quite low—never go lower and usually go higher—while anything more than 99.9% is high—never go higher and usually go lower.

IV. Optimal sample size

a. As we've discussed, there's a conflict when it comes to sample sizes. One on hand, you want them to be large (so you get a precise sample) but on the other hand, you want them to be small (to save time and money).
b. In other words, you want a sample just big enough and no larger. To figure out how big, you can use the margin of error.
c. Recall the margin of error (ME) is everything after the minus/plus sign. It's how much you're adding or subtracting from the average. If you have a desired margin of error, you can determine the minimum sample size…
d. For a standard confidence interval:

$$n = \left(\frac{z_{\alpha/2}\sigma}{ME}\right)^2$$

e. You should always round up with your sample; otherwise you won't get the desired margin of error.
f. Note we can't use an optimal sample size equation when we don't know σ. Why is that?
    i. Sample size determines degrees of freedom.
    ii. Degrees of freedom influences critical t.
    iii. Critical t influences optimal sample size.

V. CIs for Proportions

a. A proportion measures the fraction of a population, such as the percent of female viewers, the portion of customers who use a coupon, or the fraction of voters who will vote for Gandalf the Grey.
    i. Proportions cannot be less than zero or greater than 1.
b. Calculating a confidence interval for a proportion is a little different. To begin, we need to determine the standard deviation of a proportion:

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

    i. Where σ_p is the standard deviation of the proportion;
    ii. π is the population proportion; and
    iii. n is the sample size.
c. Of course, we don't know π—that's the point of the interval—but we need π to determine the interval. Our solution is the same as before:

$$\hat{\sigma}_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

    i. Where the hat is a reminder that this is an approximation based on the sample and *p-bar* is the proportion of the population.
d. Another option is to set π to 0.5; this has the effect of maximizing sigma. In other words, you have the largest standard deviation possible with your sample size. If you don't have σ from some other source, you can make your range as large as possible to ensure you "caught" the true population mean.
e. The equation for a CI should look familiar:

$$\widehat{CI}_{\bar{p}} = \bar{p} \mp z_{\alpha/2}\hat{\sigma}_p = \bar{p} \mp z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

i. Note we use the z-score here.

f. For optional sample size:

$$n = \left(\frac{z_{\alpha/2}}{ME}\right)^2 \bar{p}(1-\bar{p})$$

g. But note this equation is a little odd as it includes a value for p-bar but we don't know what p-bar is. The whole point is to set up a sample to get an estimate of p-bar.

h. One way to handle this is to look at similar work and estimate what p-bar would be. But the safest way is to set p-bar equal to 0.5. When in doubt, use this equation:

$$n = \left(\frac{z_{\alpha/2}}{ME}\right)^2 \left(\frac{1}{4}\right)$$