David Youngberg
BSAD 210—Montgomery College

LECTURE 10: CONFIDENCE INTERVALS II

I. Calculating the Margin of Error with Unknown σ

a. We often don't know σ. This requires us to rely on the sample's standard deviation, *s*. This changes everything.

b. Recall that *s* is influenced by sample size, *n*. Bigger sample sizes means we can more reasonably estimate the standard deviation. When *n* gets really large, there's no practical difference between knowing and not knowing σ.

c. But most the time, the sample is good but not enormous. Thus we cannot use our perfect normal distribution. We have to use a different distribution: Student's t-distribution.

   i. Like a normal distribution, the t-distribution is bell-shaped and symmetric around the mean.

   ii. The t-distribution is flatter and wider than the normal distribution.

   iii. The t-distribution depends on the *degrees of freedom*, or the number of values that are free to vary given that certain information is known.

      1. One bit of information that's known for any sample is the sample mean. Therefore, the degrees of freedom (df) for our use here is equal to n – 1.

   iv. The t-distribution is a family of distributions which change based on the degrees of freedom.

d. Here's the equation:

$$\widehat{CI}_{\bar{x}} = \bar{x} \mp t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

   i. The hat over CI reminds us that this is an approximation. Note how similar this equation is to the previous one.

e. Below is a table for the t-distribution with single tail alpha on top and two tail confidence levels on the bottom. Note that as df increases, the critical values approach the z-score.

Table B *t* distribution critical values

df	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

II. Critical t-values

a. Excel has a CONFIDENCE.T function for calculating the interval with a t distribution. Like CONFIDENCE.NORM, it outputs the margin of error.

b. It also has the whole table for t-values. The command for this is "=T.INV.2T"

   i. *Probability* is the alpha value, assuming two tails.

   ii. *Deg_freedom* is the degrees of freedom: n – 1.

c. So if you want to know the critical t-value for an alpha of 0.05 with six degrees of freedom, you'd type "=T.INV.2T(0.05,6)" and press ENTER.
    i. You should get about 2.447, the same as in the table under 95% confidence with 6 degrees of freedom.
    ii. Note there is a version without the 2T; the 2T stands for two tails. Without it, you're assuming alpha is only in one tail.
        1. If you think the table claims the result should be 1.943 (or more accurately it displays -1.943), it's because you're looking at the one-tailed version and not the two-tailed version. (In the two-tailed version, that 0.05 is equally split between two sides of the distribution, thus a single tail would have a value of 0.25.)

III. Example
  a. Recall Theo who works at a steel mill figured out the 95% confidence interval of scrap metal use was between 6,649.4 to 7,350.6 tons, based on an average of 7,000 tons of use from 20 weeks, with a σ (population standard deviation) of 800.
  b. Suppose he doesn't know the population standard deviation of scrap metal use per week is 800 tons. Production can vary for all kinds of reasons so he has to estimate sigma using the *sample* standard deviation, calculating it based on the sample of 20 weeks he took.
  c. ***To illustrate the effect of switching to the t distribution, assume the sample standard deviation happens to also be 800.*** Therefore, the only think that changes in the calculation is we're using t scores instead of z scores.
  d. Using CONFIDENCE.T(0.05,800,20), we get a new margin of error: 374.4 tons, higher than the 350.6 we found earlier. This higher margin of error reflects that we have less information than before. Because we don't *know* σ, we have to *estimate* σ, and that's not as accurate.
  e. Note this distinction shrinks as n increases because the amount of information we have increases. Thus the sample standard deviation (s) is likely to be close to population standard deviation (σ). To illustrate, suppose n was 200 weeks instead of 20 weeks.
      i. CONFIDENCE.NORM(0.05,800,200) = 110.9
      ii. CONFIDENCE.T(0.05,800,200) = 111.6
      iii. Switching to the t distribution when n was so high made the margin of error only 0.6 percent larger (before it was 6.8 percent larger).

IV. CIs for Proportions

a. A proportion measures the fraction of a population, such as the percent of female viewers, the portion of customers who use a coupon, or the fraction of voters who will vote for Gandalf the Grey.

   i. Proportions cannot be less than 0 or greater than 1.

b. The equation for a CI should look familiar:

$$\widehat{CI}_{\bar{p}} = \bar{p} \mp z_{\alpha/2}\hat{\sigma}_p = \bar{p} \mp z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

   i. Where *p-bar* is the sample proportion of the population; and

   ii. *n* is the sample size.

   iii. Note we use the z-score here.

V. Example

a. A polling company wants to know what percent of voters will support a new ballot initiative in the upcoming election. They interview 500 voters and find 56% support the initiative. At 99% confidence, what is the confidence interval of the poll?

b. Sadly, Excel does not have this equation built in (yet) so we have to do this the old fashion way. Set up Excel like this:

	A	B
1	**p-bar**	0.56
2	**n**	500
3	**z**	2.576
4		
5	**MoE**	
6		
7	**LB**	
8	**UB**	

   i. Where MoE is the margin of error, LB is the lower bound, and UB is the upper bound.

   ii. In B5, we do this: =B3*SQRT(B1*(1-B1)/B2)

   iii. Keep in mind that SQRT is "square root." We are taking the square root of 0.56 times 0.44 divided by 500. Then we multiply the whole thing by the z score. You should get about 0.0572.

   iv. For the lower and upper bounds, add and subtract from p-bar, as you've done previously with x-bar:

	A	B
1	**p-bar**	0.56
2	**n**	500
3	**z**	2.576
4		
5	**MoE**	=B3*SQRT(B1*(1-B1)/B2)
6		
7	**LB**	=B1-B5
8	**UB**	=B1+B5

v. So you should get:

	A	B
1	**p-bar**	0.56
2	**n**	500
3	**z**	2.576
4		
5	**MoE**	0.0572
6		
7	**LB**	0.5028
8	**UB**	0.6172

c. There's a 99% chance that the true support of voters is between about 50.3% and 61.7%.

VI. Additional Notes on Confidence Intervals

a. Choosing a confidence level is tricky, which is why sometimes you see multiple levels reported. That's not always the case, though, because—bizarrely—people often want definite answers when it comes to statistics.

i. On one hand, a narrow range tells you a lot about what the population mean might be. You are more precise but risk completely missing the mark.

ii. On the other hand, a wider range ensures the population mean is in that band. Your range probably includes the parameter, but you're vague.

iii. The question is, what side is best to err on? That changes with circumstance. That said, 95% confidence is quite low: never go lower and usually go higher.

iv. Sometimes you see people refer to a 90% level. I used to give those z values but took them out because 90% is frankly pretty desperate.