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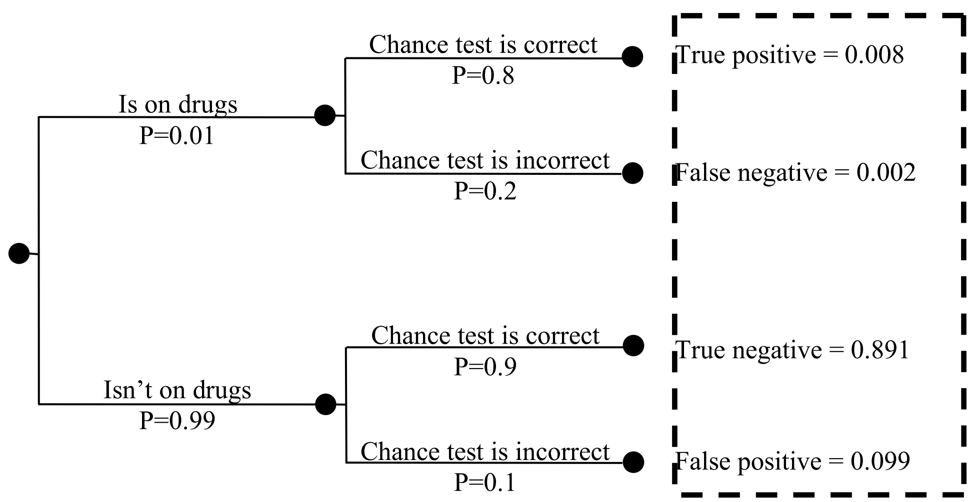
LECTURE 33: BAYES' THEOREM II

I. Tree Diagram, Revisited

a. Now let's calculate the chance of each scenario. We do this by multiplying each conditional probability by the appropriate condition.

i. Recall for a true positive to occur, the subject must be on drugs AND the test must come back positive.



b. Note there are two ways to get a positive result (true and false) and two ways to get a negative result (true and false).

i. Chance of a positive result: 0.008 + 0.099 = 0.107 = 10.7%

ii. Chance of a negative result: 0.002 + 0.891 = 0.893 = 89.3%

c. Now the really interesting part: if you get a positive result, what are the chances the person is actually a drug user?

i. To answer, determine what portion of positive results are true positives:

Chance of a true positive ↘

$$\frac{0.008}{0.107} \cong 0.0748, or\ 7.48\%$$

Chance of a positive result ↗

ii. Before testing, you had a 1% chance of being a user. After testing, a positive result increased your likelihood of being a user but because the condition (drug user) is so rare, the very many false positives overwhelm the true positives.

iii. In other words, it's hard to detect rare things.

II. Conditional probability and Bayes' Theorem

a. This can all be summarized with *Bayes' Theorem*—a way to describe how one should adjust their beliefs to account for evidence.

b. To better understand Bayes' Theorem, let's first understand some terminology.

i. *P(A)*—the probability event A will occur.

ii. *P(A|B)*—the probability event A will occur assuming event B happens; this is the conditional probability.

iii. ~ —this symbol means "not." P(~A) means "the probability event A will not occur." Thus P(A) + P(~A) = 1.

iv. Employing the example from before, let's let **U** be "employee is a user" (so ~U is "employee is not a user") and **+** be "the test came back positive for drugs" (so ~+ is "the test is not positive, or came back negative for drugs").

c. Let's begin with an important equivalency.

$$P\,(B) = P(B|A)\,P(A) + P(B|{\sim}A)\,P({\sim}A)$$

i. Why does this work? If you're trying to figure out how likely is it for B to happen, recognize that there's a chance of B happening if A happens and there's a chance of B happening if A doesn't happen. Adjusting for the likelihood A happens, and you get the probability that B happens.

ii. For example, what is the chance of you having a good job in five years? You can have a good job if you get a bachelor's degree and you can have one if you don't get a bachelor's degree but the chance of the first conditional is much higher than the chance of the second conditional.

d. Bayes' Theorem is as stated:

$$P(A|B) = \frac{P(B|A)\,P(A)}{P\,(B)} = \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|{\sim}A)\,P({\sim}A)}$$