

LECTURE 26: COMPARING TWO POPULATIONS III

I. Dependent samples

- a. Now we turn to when each observation in one sample is related to an observation in the other sample: a *dependent sample*.
- b. A dependent sample occurs when you can match two different observations from two different samples based on a common subject. This is called a *matched-pair test*.
 - i. Examples: customer behavior before and after a promotion; a person's weight before and after a diet; store performance when an additional person is hired; etc.
 - ii. Note many of these examples involve the same subject with before and after measurements. But it could also involve testing the same subject while under one new condition and then testing the same subject under the new condition with different circumstances.

II. Mathematics

- a. In this analysis, each observation in a sample has a “partner” in the other sample. We're interested in the difference between each pair, called d .
 - i. $d = x_1 - x_2$
- b. Adding all these differences and dividing by the sample size results in \bar{d} , the average matched-pair difference.
 - i. This means that the number of observations from each sample must be equal.

$$t_{\bar{x}} = \frac{\bar{d} - (\mu_d)_{H_0}}{\frac{s_d}{\sqrt{n}}}$$

- ii. Where s_d is the standard deviation of the differences; and
 - iii. \bar{d} is the average difference of the matched-pairs.
- c. Suppose you're testing a weight-loss pill. You weigh 16 randomly selected people. The average weight change is -2.6 pounds and the standard deviation is 5. Note this is a one-tailed test.

$$t_{\bar{x}} = \frac{\bar{d} - (\mu_d)_{H_0}}{\frac{s_d}{\sqrt{n}}} = \frac{-2.6}{\frac{5}{\sqrt{16}}} = \frac{-2.6}{\frac{5}{4}} = \frac{-2.6}{1.25} = -2.08$$

- ii. Where s_d is the standard deviation of the differences; and
 - iii. \bar{d} is the average difference of the matched-pairs.
- d. With 15 degrees of freedom, the critical value at $\alpha=0.05$ is 1.753 so it's statistically significant. It's barely shy the 2.131 value at $\alpha=0.025$.

III. Finished Diagram

