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LECTURE 22: AVOIDING TYPE II ERRORS I

I. Scenario
   a. Imagine the average electric car battery lasts 100 miles before needing to be recharged. The standard deviation is 12 miles.
   b. Suppose you create a battery which you think lasts longer.
      i. H_0: $\mu \leq 100$ miles
      ii. H_a: $\mu > 100$ miles
   c. Finally, imagine you tested your battery on fifty cars and found the average distance traveled to be 105 miles. Is your battery good?
   d. As we learned, we can find the z-score:

$$z = \frac{105 - 100}{12/\sqrt{50}} = \frac{5}{12/\sqrt{50}} = \frac{5}{1.697} = 2.946$$

      i. At 95% confidence (z=1.645, one-tail), it's statistically significant.
      ii. How low could our average be and still be significant?

$$\bar{x}_\alpha = \mu_{H_0} \pm z_\alpha \left(\frac{\sigma}{\sqrt{n}}\right)$$

      iii. The plus/minus sign indicates that we must decide to add or subtract depending on what kind of test it is. Because we're interested in the battery lasting *longer*, we must add. The *critical sample mean* (x-bar$_\alpha$), or the sample mean that marks the boundary of rejection, will be on the right side of the distribution.
      iv. At 95% confidence,

$$\bar{x}_\alpha = 100 + 1.645\left(\frac{12}{\sqrt{50}}\right) = 100 + 1.645(1.697) = 102.8\,miles$$

      v. Even if we had a difference of just below three miles, it would still be statistically significant. (It is of course a different question if it is of practical significance.)

II. Alpha and Beta

a. Alpha represents the probability that an extreme value is a coincidence. It's the chance we're making a Type I error.
    i. If we make a Type I error, the null hypothesis is true and our sample mean is unusually large or small by coincidence (represented by α). We reject the null when we should have failed to reject it.
b. Type I error is not the only kind of error we can make; Type II error exists as well. It is a sneaky kind of error.
    i. We represent the chance of a Type II error with β.
    ii. Type I and Type II error are inherently linked. One α is set, β is locked in place (assuming sample size is not changed).
        1. Increasing α decreases β and vice versa. The higher your standard for making sure you avoid a Type I error, the more likely you'll not meet that standard by chance and make a Type II error.
        2. Increasing sample size decreases both α and β.
c. In other words, we run an experiment on new technology involving battery life and the sample average is not much longer than the current state of technology.
    i. Since it's not statistically significant, we claim our technology doesn't work. But we could be making an error; maybe our sample happened to include particularly low values.
    ii. Remember, the Central Limit Theorem is always there.