David Youngberg
BSAD 210—Montgomery College

LECTURE 20: A TALE OF TWO TAILS

I. Let's revisit that list from an earlier lecture. Notice the expansion.

Confidence	α	$z_{\alpha/2}$	z_{α}
90%	10%	1.645	1.280
95%	5%	1.960	1.645
99%	1%	2.576	2.330
99.9%	0.1%	3.291	3.090

   a. The additional column is for when the significance level is concentrated on just one side of the distribution.
   b. This brings up the difference between a one-tail hypothesis test and a two-tail hypothesis test.
      i. In a *one-tail hypothesis test* the alternative hypothesis is stated with a "<" or a ">" and the null hypothesis is stated with a "≥" or a "≤", as appropriate.
      ii. In a *two-tail hypothesis test* the alternative hypothesis is stated with a "≠" and the null hypothesis is stated with a "=".
   c. There is no difference in equation when you consider a one-tail or two-tail test. The only difference is the significance levels.
      i. You can use the t-distributions to tell you one- or two-tail values. Note in the table above, the one-tail z-score at 95% is identical to the two tail score at 90%. That's because in both cases the number of observations under <u>one</u> tail is 5%.

II. One-Tail or Two?
   a. The question then becomes: when should you use one-tail or two-tails? This involves answering a different question: what do people care about?
   b. One-tail tests are best for claims of improvement, where doing worse is effectively the same as doing average; neither is impressive. One-tail tests also used to refute points of view (someone might say something is popular so the alternative is that it's not popular).
      i. <u>Examples</u>: Longer battery life; faster acceleration time; the popularity of gay marriage.
      ii. One-tail tests are great because you get to claim a higher confidence level with the same z-score. If, for example, your z-

test is 1.7, you're significant at the 95% level but for a two-tailed test, you'd only be significant at the 90% level.

c. Two-tail tests are best for when you're trying to detect unusualness in either direction. In other words, there's a "sweet spot" that the null hits but the alternative doesn't. The question is *not* if the value is more or less than the average but if the value is *different*.
    i. Examples: the accuracy of a machine putting ketchup in a bottle (could be putting too much or too little in); the length of time you must stand in line at a store (both too little or too much is noteworthy); if a new employee is unusually good or bad at the job.
    ii. Because a two-tailed test has a higher standard than a one-tailed test, it's what you use when you're not sure.