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LECTURE 17: CONFIDENCE INTERVALS I

I. The Meaning of Confidence

a. When we take a sample, we often summarize it with the sample's mean. This is an example of a *point estimate*—a single value that best describes the population.

b. The problem is, our sample may be unusual. Remember the Central Limit Theorem? Sample means form a normal distribution. Our sample mean, and therefore our sample, could be unusual.

c. We correct this with a confidence interval. A *confidence interval for the mean* is the range where the true population mean lies.

   i. For example, the confidence interval for how customers feel about your restaurant is between 7.4 and 8.1.

d. The *confidence level* is the probability that the interval estimate will include the population parameter of interest (e.g. the population mean).

   i. For example, 90% confidence means that if you took 100 samples, 90 of those samples' means would lie within the confidence interval.

   ii. "Confidence" in statistics is not an emotional state. It's a reference to the probability of where the true parameter lies.

II. Z-scores

a. Z-scores are critical to confidence intervals. Indeed, we call them *critical values* because that is the threshold value. For example, what should have a larger range: 90% confidence or 99% confidence?

   i. Both are useful. 90% confidence narrows the choices of where the true parameter lies but 99% confidence will have a larger range and thus you'll be more likely to know where the true value is.

   ii. The question becomes, how do you mathematically capture that change from 90% to 99%?

b. Z-scores are standard deviations. That means at a z-score of 2, 95% of observations are within two standard deviations. Every confidence rating has a z-score associated with it.

   i. Technically, 95% confidence is a z-score of 1.96.

c. The Greek letter α (alpha) is the *significance level*; it's equal to 1 – confidence level. For example, at 90% confidence, $\alpha = 0.10$.

i. Because there are two sides of the normal distribution, α is sometimes divided by two to indicate the area for alpha is split in half: α/2.

ii. Every critical value has a corresponding z-score, $z_{\alpha/2}$. Below is a table of regularly used z-scores.

Confidence	*α*	$z_{\alpha/2}$
90%	10%	1.645
95%	5%	1.960
99%	1%	2.576
99.9%	0.1%	3.291

iii. As a life skill, it's useful to memorize this list even if you only memorize it out to two decimal places rather than three.

III. Calculating the Margin of Error with Known σ

a. The *margin of error* is the width of a confidence interval: the distance between an upper or lower bound and the mean.

i. Because the normal distribution is symmetric, the distance between the upper bound and the sample mean equals the distance between the lower bound and the sample mean.

b. When the population standard deviation (σ) is known, the equation for a confidence interval is this:

$$CI_{\bar{x}} = \bar{x} \mp z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

i. Where $CI_{x\text{-bar}}$ is the confidence interval for the sample mean;

ii. *x-bar* is the sample mean;

iii. $z_{\alpha/2}$ is the critical value for α significance level;

iv. σ is the population's standard deviation; and

v. *n* is the sample size.

vi. Note the minus/plus sign means you subtract (to the get the lower bound) and add (to get the upper bound).

vii. The margin of error is everything after the minus/plus sign.

c. You typically know σ concerning well-established data. This includes blood pressure, stock market prices, or any other data that's calculated regularly and for a long period of time.

d. But knowing σ means you know μ (recall you need μ to calculate σ) so a confidence interval isn't as useful here.

        i. Sometimes μ changes and we have good reason to think σ didn't change—like shifting the entire normal distribution right or left—so this there's still a use for what we just covered.

IV. Calculating the Margin of Error with Unknown σ

   a. We often don't know σ. This requires us to rely on the sample's standard deviation, *s*. This changes everything.
   b. Recall that *s* is influenced by sample size, *n*. Bigger sample sizes means we can more reasonably estimate the standard deviation. When *n* gets really large, there's no practical difference between knowing and not knowing σ.
   c. But most the time, the sample is good but not enormous. Thus we cannot use our perfect normal distribution. We have to use a different distribution: Student's t-distribution.
      i. Like a normal distribution, the t-distribution is bell-shaped and symmetric around the mean.
      ii. The t-distribution is flatter and wider than the normal distribution.
      iii. The t-distribution depends on the *degrees of freedom*, or the number of values that are free to vary given that certain information is known.
         1. One bit of information that's known for any sample is the sample mean. Therefore, the degrees of freedom (df) for our use here is equal to n – 1.
      iv. The t-distribution is a family of distributions which change based on the degrees of freedom.
   d. Here's the equation:

$$\widehat{CI}_{\bar{x}} = \bar{x} \mp t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

      i. The hat over CI reminds us that this is an approximation. Note how similar this equation is to the previous one.

   e. Below is a table for the t-distribution with single tail alpha on top and two tail confidence levels on the bottom. Note that as df increases, the critical values approach the z-score.

Table B t distribution critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											