

## LECTURE 15: DISCRETE PROBABILITY FUNCTIONS II

### I. Hypergeometric Distributions

- a. In a binomial distribution, we discussed the assumption of each trial as independent. For example, the sample taken is less than 5% of the population or there is replacement.
- b. What happens if your sample is more than 5% and there's no replacement? If each trial affects the likelihood of success, we need to use a different discrete probability function: hypergeometric.
- c. The mean is:

$$\mu = \frac{nR}{N}$$

- i. Where  $N$  is the population size
  - ii.  $R$  is the number of successes in the population
  - iii.  $n$  is the sample size.
- d. The standard deviation is:

$$\sigma = \sqrt{\frac{nR(N-R)}{N^2}} \sqrt{\frac{N-n}{N-1}}$$

- e. The probability of success is as follows:

$$P(x, n) = \frac{{}_{N-R}C_{n-x} \times {}_R C_x}{{}_N C_n}$$

- i. Where  $x$  is the number of successes.

### II. Poisson Distribution

- a. This type of distribution describes the number of times some event occurs during a particular interval (such as time, distance, area, volume, etc). Unlike other distributions, there can be any number of occurrences (successes).
  - i. Examples: number of returns in an hour; number of strawberries in a patch that don't pass quality control; number of lost golf balls per year at a mini-golf course.

- b. Requirements
  - i. Mean must be the same for each interval.
  - ii. Intervals cannot overlap.
  - iii. Occurrences in each interval must be independent.
- c. If we know how often something occurs on average, we can use Poisson to figure out how often something other than the average occurs.
  - i. Because the Poisson distribution begins with knowing the average number of events, there is no equation for the average number of events.
- d. The standard deviation is:

$$\sigma = \sqrt{\lambda}$$

- i. Where  $\lambda$  is the average number of events that occur in the period in question.
- e. The probability of exactly  $x$  events occurring is:

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{\lambda^x}{x! e^\lambda}$$

- i. Where  $e$  is about 2.718
      - ii. Note that the negative exponent means that part of the equation goes to the denominator.