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LECTURE 13: BAYES' THEOREM II

I. Bayes' Theorem

a. Before testing, you had a 1% chance of being a user and a 99% of not being a user. Note that, after testing, a positive result increased your likelihood of being a user and decreased your likelihood of not being a user.

b. This is an example of *Bayes' Theorem*—a way to describe how one should adjust their beliefs to account for evidence.

c. To better understand Bayes, let's first understand some terminology.

  i. *P(A)*—the probability event A will occur.

  ii. *P(A|B)*—the probability event A will occur assuming event B happens.

  iii. ~ —this symbol means "not." P(~A) means "the probability event A will not occur."

  iv. Employing the example from Part II, let's let **U** be "employee is a user" (so ~U is "employee is not a user") and **+** be "the test came back positive for drugs" (so ~+ is "the test is not positive, or came back negative for drugs").

d. Bayes' Theorem is as stated:

$$P(A|B) = \frac{P(B|A)\,P(A)}{P\,(B)} = \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|{\sim}A)\,P({\sim}A)}$$

  i. Note why the denominators are equivalent. If A happens, how likely is B? If A doesn't happen, how likely is B? Adjusting for the likelihood A happens, and you get the probability that B happens. This last step is similar to how we solved the Simpson Paradox; it's a weighted average.

e. We want to know "what is the probability that someone who tested positive for drugs is actually a user?" Or, given that the test is positive, how likely is it that they actually use drugs? Or, what is P(U|+)?

$$P(U|+) = \frac{P(+|U)\,P(U)}{P(+)} = \frac{P(+|U)\,P(U)}{P(+|U)\,P(U) + P(+|{\sim}U)\,P({\sim}U)}$$

$$= \frac{0.9 * 0.01}{0.9 * 0.01 + 0.1 * 0.99} = \frac{0.009}{0.009 + 0.099} = \frac{0.009}{0.108} \cong 0.083$$

i. Note the result is as before: 8.3%. Only 8.3% of positive results are actually drug users!

f. Bayes' Theorem thus tells us two interrelated things:

i. When you receive new information (such as the results of a drug test), it tells you how much to adjust your estimation of the truth.

ii. It reminds you that because no test is 100% accurate, its results should not be weighed too heavily, especially if it is testing for something rare (since the false positives will overwhelm the true positives).