

LECTURE 10: PROBABILITY

- I. Probability
 - a. Express probability as a decimal or fraction—the likelihood something will occur is never greater than 1.
 - b. “Mutually exclusive” means you can add the probabilities.
 - i. If both events can’t occur at the same time (mutually exclusive), “or” means you add them as well.
 - ii. Otherwise, add them and then subtract the product of the two.
 - iii. Example: Probability of drawing either a queen or a heart card isn’t $(4/52) + (13/52)$ because you have to include the card that’s both. What’s the actual probability?
 - c. “Independent” means you can multiply the probabilities.
 - i. If the outcome of one event doesn’t affect the likelihood of the other (independent), “and” means you multiply them.
 - ii. Otherwise, multiply by the conditional probability (see below).
 - iii. Example: Probability of drawing two aces isn’t $(4/52)(4/52)$ because if you draw one, there’re only three aces left. What’s the actual probability?
 - d. One of the most interesting applications of probability is the Monty Hall paradox.
- II. Monty Hall paradox
 - a. Suppose before you were three doors. Behind one of these doors is a car. Behind each of the other two is a goat. Pick a door.
 - b. Before the door’s opened, suppose someone opens one of the two other doors. This person always opens a door with a goat behind it.
 - c. You are then offered to change your selection. Do you? Does it matter?
 - d. Here’s the paradox: it doesn’t *seem* to matter what you do, but it matters a great deal. The strategy here is always the same: you switch.
 - i. When I first heard this paradox, I didn’t think it would matter. It took a while before I believed it did.
 - ii. By the way, the name of this paradox comes from a game show starring Monty Hall. He put contestant after contestant in this scenario.
 - e. *Conditional probability* is the probability one event will occur given another event has occurred.

- i. The probability I'm carrying an umbrella on any random day is low. But the probability I'm carrying an umbrella given it rained today is much higher than the nonconditional. The probability I'm carrying an umbrella given that it snowed today is much lower than the nonconditional.
- f. Suppose you always swap:
 - i. **If** you pick a car the first time, swapping means you never get the car.
 - ii. **If** you pick a goat the first time, swapping will get you always get the car.
- g. Suppose you never swap:
 - i. **If** you pick a car the first time, staying means you always get the car.
 - ii. **If** you pick a goat the first time, staying means you never get the car.
- h. Since the probability of picking the car is ____% and the probability of picking the goat is __%, swapping is always the better idea.
 - i. This is because the host eliminated one of the undesirable choices for you; he gave you information.
 - ii. Still don't believe me? Then let's look at a series of tests.
http://www.youtube.com/watch?v=o_djTy3G0pg