David Youngberg
BSAD 210—Montgomery College

LECTURE 07: GEOMETRIC MEAN

I. Geometric Mean
   a. When taking the average of growth rates, it's helpful to calculate the average differently. To understand why, consider the annual sales growth rate of a company. Last year it was 1% and the year before that it was 9%. If sales started at $100,000, what are the sales now?
      i. A 9% increase in sales means sales grew by $9,000; it became $109,000.
      ii. A 1% increase in sales means sales grew by $1,090; it became $110,090.
      iii. In other words, sales went from $100,000 to $110,090. Or:

      ($100,000)(1.09)(1.01) = $110,090

      Note the use of adding "1" to the growth rate. That way we not only include what's being added but also what we started with.
   b. We can simplify the approach with this equation:

$$New\ result = Starting\ amount \times \prod_{i=1}^{n}(1 + x_i)$$

      i. The giant pi symbol means multiply;
      ii. The "x's" are the growth rates, expressed as a decimal;
      iii. The "i" means you're considering the ith rate;
      iv. The "N" means there are that many rates to consider.
      v. In our example, N was two, x_1 was 0.09 and x_2 was 0.01.
   c. Now suppose we claimed the average growth rate was 5%. That means if the growth was five each year, we should get the same total sales. But we don't.
      i. ($100,000)(1.05)(1.05) = $110,250.
      ii. We got a higher number than before. It may seem close enough, but keep in mind it should be *exactly the same* and we were only using two years. If you repeated this example using ten or twenty years of data, we'd be way off.

iii. Using the "arithmetic mean" on growth rates results in overstating the average growth rate. We have to use the geometric mean.

d. Here's the equation for the geometric mean:

$$Geometric\ Mean = \sqrt[n]{\prod_{i=1}^{n}(1 + x_i)} - 1$$

i. Rather than adding all the observations up and dividing by the number of observations, we're multiplying all the observations together and then taking the Nth root. Note how similar this is

ii. So our growth rate is:

$$Geometric\ Mean = \sqrt[2]{(1.09)(1.01)} = \sqrt[2]{(1.1009)} \cong 1.0492 - 1 = 0.492$$

iii. A more accurate growth rate would be just over 4.92%.

II. Geometric Mean Practice

a. Here's a video of this section (the technique is slightly different).

b. Suppose we're also curious about the company's average growth rate. To find the geometric mean, recall the best way is to add 1 to all observations, take the geometric mean, and then subtract one.

i. In P5, type "1+O5". Because all the growth rates are displayed as percents, you should get 1.02. You can increase the decimal places displayed with the button in the Number section. But you don't have to do this; Excel knows those values are there even if they are not shown.

ii. Now double-click the square in the lower-right corner of the selected box.

iii. In P23, type "=GEOMEAN(P5:P21)-1". You may want to click the % button to the left of the decimal button and increase the decimal. You should get about 4.67%.

iv. Note if we took the arithmetic mean, you'd get about 4.75%. That doesn't sound like much of a difference, but if you assumed 4.75% growth every year starting in 1998, you'd get about $600 million more in sales than you'd actually have. But with 4.67%, you are exactly correct.

c. You can also use "=SUMPRODUCT(GEOMEAN(1+ O5:O21))-1"