David Youngberg
BSAD 210—Montgomery College

Lecture 21: Expected Value

I. Expected value
   a. Suppose I roll a four sided die and pay you a dollar if the result is even and nothing if the result is odd. How much are you willing to pay to play this game?
      i. What if you only get paid if the result is a one?
      ii. Or suppose I pay you, in dollars, the result of the die? (Rolling a "2" gets you $2; a "4" gets you $4.)
   b. What game would you rather play?
      i. If the die is even, you lose $1 and if odd, you get $1.
      ii. If the die is even, you lose $10 and if odd, you get $10.
   c. Both questions can be aided by *expected value*, or the value of a random payoff. (Expected value is also called the mean, or average).
      i. To determine expected value, multiply the payoff of each scenario by the probability of that scenario happening. Then add these values together.
      ii. So for the game in I.a.ii., the expected value is:

$$\frac{1}{4}\$1 + \frac{1}{4}\$2 + \frac{1}{4}\$3 + \frac{1}{4}\$4 = \$2.50$$

II. Attitudes towards risk
   a. We have not yet answered the question from the beginning: how much are you willing to play these games?
   b. Just because we know the expected value, doesn't mean we know how much people are willing to pay. The very idea of a risky payoff can be exciting for some and nerve wrecking for others.
   c. It's useful to compare expected value, E(x), with a certain amount of A, where A = E(x).
      i. *Risk loving* individuals prefer E(x) to A.
      ii. *Risk neutral* individuals are indifferent to E(x) and A.
      iii. *Risk averse* individuals prefer A to E(x).
   d. Different people have different risk preferences and these preferences can vary for the same person from time to time. Some industries are built on how people approach risk.

e. The insurance industry is built around people being risk averse. People find more pain in facing –E(x) compared to –A, where –A is the insurance price and –E(x) is the expected cost of an accident. They would rather lose A than lose E(x).
    i. Suppose each person has a 1% chance of having an auto accident which causes $1,000 in damages, or an expected cost of $10. Suppose each person is willing to pay $15 to get insurance.
    ii. With, say, 50,000 customers, the insurance company receives $750,000 in revenue and pays out only $500,000 in damages, leaving room left over to pay for the costs of business and secure a healthy profit.

f. The gambling industry is built around people being risk loving. People prefer E(x) to A, where A is the cost to play a game and E(x) is the expected payoff from that game. They will give up A to get E(x).
    i. Roulette is a game where you spin a disc with 38 spaces and a spinning ball that eventually lands on one of those spaces. If it lands on a space of yours, you get 35 times what you paid in (one dollar becomes $35), and lose the dollar otherwise.
    ii. Since the probability of success is 1/38, the expected value of winning is $35/38, or $0.92. The probability of losing is 37/38 so the expected value of losing is -$0.97. Added together, every player of roulette loses, on average, five cents per dollar gambled; the casino gets five cents. If $100,000 are gambled on roulette, the casino walks away with $5,000 and the customers leave $5,000 poorer.

g. When it comes to income, most people are risk averse. They prefer a steady income than a fluctuating one. Remembering compensating differentials, jobs with a fluctuating income should pay higher, on average, than those with a steady income.

h. Note that a person's attitude towards risk is ***not*** the same as her attitude towards uncertainty.
    i. People tend to be either risk loving, risk neutral, or risk averse. We compare a riskless payoff to a probabilistic one to see.
    ii. But people tend to be averse to ambiguity. They prefer the probabilistic payoff to an unquantifiable payoff.

III. Construction Example

a. Let's revisit another example from the last set of topic notes: construction's time to completion.

b. We have a nice table indicating the various times it will take to complete a project:

Total Time	Probability
11 months	0.008
12 months	0.076
13 months	0.264
14 months	0.412
15 months	0.240

c. But customers don't always want a range. Sometimes they want a clear, single number. We use what we've learned to find that number.
d. Simply multiply each probability by the appropriate completion time. Then add them together.
    i. Note this is a weighted average: the probabilities are the weights. And because the total probabilities add up to one, there is no need for an additional step of division.
e. So let's multiply and add:

Total Time	Probability	Expected
11 months	0.008	0.088
12 months	0.076	0.912
13 months	0.264	3.432
14 months	0.412	5.768
15 months	0.240	3.60
Expected Time (months)		**13.8**

f. A better estimation is a little less than 14 months: 13.8, or about 13 months and 3 weeks.

IV. Prediction markets
a. Talk is cheap. Saying something will happen without any punishment if you're wrong means little. Betting on something says a lot more.
    i. Mere talk means it is cheap to indulge sloppy thinking like bias and poor logic. If you're wrong, nothing happens.
b. If we allow people to "bet" on future events, we get a more balanced assessment. People put their money where their mouth is.
c. If we allow many people to bet, we can create a market. With the market, we'll have prices which we can use to inform us about the future.
d. *Prediction market*—speculative market designed so that prices can be interpreted as probabilities about future events.

i. Suppose you buy "tickets" with some event on them (e.g. Hilary Clinton is elected President). If the event happens by an indicated date, the ticket is worth $1.00 (or $10, or $100). If not, it is worth zero.
ii. This is expected value in action. But here, the expected value, the market price, is given to us. We know the payoff if the event happens (what the ticket is worth). We solve for the probability.

e. Predicitit.org values its tickets at $1 each. As of January 14, 2017, you could buy a ticket that Cory Booker would run for president in 2020 at a cost of 59 cents.[1] That means he has a 59% chance of running.

V. Negligence

a. When employees or customers are harmed by something the firm could have prevented, should the firm be punished? All the time?
i. Some accidents are really bad and others only cause a little harm.
ii. Some accidents have a high chance of happening and others are very rare.
iii. For common and dangerous accidents, it's reasonable the firm should work hard to prevent them; the expected cost is high.
iv. For rare and minor accidents, it's not reasonable that the firm should work hard to prevent them; the expected cost is low.
v. Expected cost teaches us that there's no inherent difference between the other two. A rare but bad accidents has the same expected cost as a common but minor accident.
vi. Here's a table of the expected costs:

	Harm is high if accident happens	**Harm is low if accident happens**
Accident is common	High	Moderate
Accident is rare	Moderate	Low

b. This if further complicated by what the firm must do to prevent the accident. Some measures are costlier than others. But prevention must happen *before* the accident can happen. Otherwise, there's no point to the prevention method. It's not enough to think about the expected cost; you must also consider how hard it would be to prevent such a thing from happening. This brings us to the Learned Hand Formula.

[1] https://www.predictit.org/Contract/5120/Will-Cory-Booker-run-for-president-in-2020#data

c. Crafted by Judge Learned Hand in 1947, the Learned Hand Formula describes someone should be held responsible due to negligence if:

$$B < pL$$

   i. Where **B** is the burden of avoiding the accident,
   ii. **p** is the probability the accident will occur, and
   iii. **L** is the cost of the accident.

d. So if there are no handrails (which are cheap to install) to prevent people from falling off a balcony (which is common **and** dangerous), the owner will be held liable.

e. But if an owner didn't clear the sidewalk of ice shortly after it formed (which is expensive to do) to prevent people from slipping (uncommon given the time constraint **and** not very harmful if it happens), the owner won't be held liable.

f. Consider the 9/11 terrorist attack.
   i. Such an attack is very, very rare (p is low).
   ii. But the cost of it happening is very, very high (L is large).
   iii. Is it the airlines' fault? A stronger door to the cockpit would have stopped it, but you would have to base that additional cost on replacing cockpit doors on *all* the planes (because you don't know which ones will be hijacked before hand). And would installing these doors make other things more difficult (if such doors get stuck, it creates a BIG problem if they can't be knocked down).
   iv. It's a tricky question, but the formula helps you approach the problem in a systematic way. Knowing this is a common standard can also help prevent you being successfully sued in the future.[2]

g. This formula came from the case United States v. Carroll Towing Co. (1947). You can read about it here.

[2] While I am aware of this negligence standard, I am not a lawyer. Always consult with a professional first.