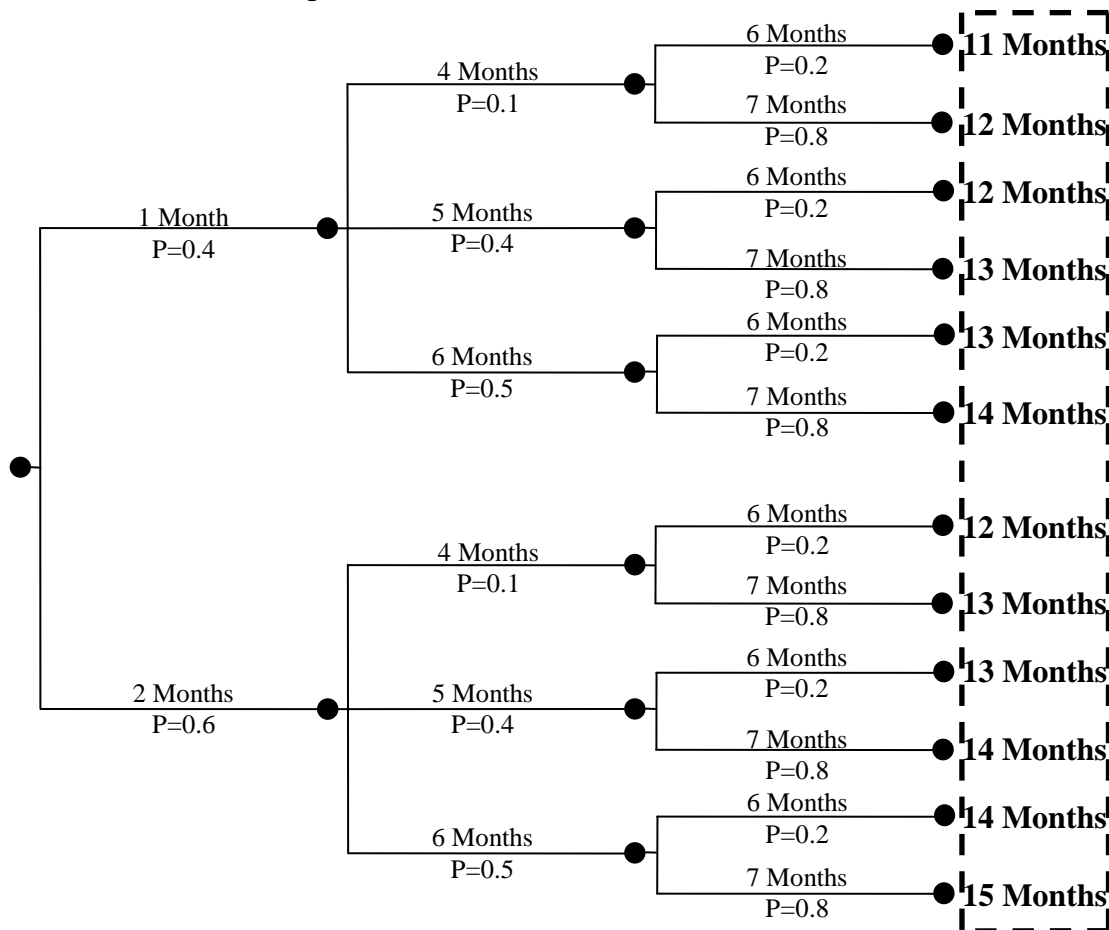


LECTURE 20: PROBABILITY

- I. Probability
 - a. Express probability as a decimal or fraction; the likelihood something will occur is never greater than 1.
 - i. 20% becomes 0.2
 - ii. 1% becomes 0.01
 - iii. 0.5% becomes 0.005
 - b. “Mutually exclusive” means you can add the probabilities.
 - i. If both events can’t occur at the same time (mutually exclusive), “or” means you add them as well.
 - ii. Because each outcome of all possible outcome mutually exclusive, adding the probability of all possible outcomes results in a sum of 1.
 - iii. Example: Probability of drawing either a queen *or* a heart card isn’t $(4/52) + (13/52)$ because you have to include the card that’s both.
 - c. “Independent” means you can multiply the probabilities.
 - i. If the outcome of one event doesn’t affect the likelihood of the other (i.e. independent), “and” means you multiply them.
 - ii. Because you’re introducing more requirements for a success, the result should go down. And because no probability is greater than one, the result does go down.
 - iii. Example: Probability of drawing an ace *and* then another ace isn’t $(4/52)*(4/52)$ because if you draw one, there’re only three aces left.
 - d. That all possible outcomes equals 1 allows you to find other probabilities. For example, the likelihood of drawing a heart card is 0.25 ($13/52$); the likelihood of drawing a non-heart card is thus 0.75 ($1 - 0.25$).
- II. Tree Diagram
 - a. Business decisions often come with uncertainty on many levels. One way to capture all what’s going on is with a tree diagram—a graphical representation of all possible outcomes.
 - b. Consider a major construction project with occurs in three stages: demolition of the old building, parking garage which makes up the foundation, and the construction of the high-rise apartment building.

- c. At each stage, delays can occur. Suppose:
- Demolition has a 40% chance of taking 1 months and a 60% chance of taking 2 months.
 - Foundation has a 10% chance of taking 4 months, 40% chance of taking 5 months, and a 50% chance of taking 6 months.
 - Building has a 20% chance of taking 6 months, 80% chance of taking 7 months.
- d. Suppose each stage is independent; if it takes longer on Demolition that means it won't affect Foundation's probabilities.
- e. Our tree diagram "maps" every path the construction can take, with probabilities indicated. The total time to completion is in the box on the far right.



- f. The project will take anywhere from 11 to 15 months. But some time frames are more likely than others.
- How likely is the first scenario? The demolition happens smoothly AND the foundation is poured without a problem AND everything happens as expected when the building's built.

To determine how likely all these things happen, multiply the probabilities: $(0.4) \cdot (0.1) \cdot (0.2) = 0.008$; in other words, a 0.8% chance. Note that this multiplication is allowed because of what I mentioned in part d.

ii. Let's do the same thing for all scenarios:

Scenario	Total Time	Demolition Probability	Foundation Probability	Building Probability	Overall Probability
1	11 months	0.4	0.1	0.2	0.008
2	12 months	0.4	0.1	0.8	0.032
3	12 months	0.4	0.4	0.2	0.032
4	13 months	0.4	0.4	0.8	0.128
5	13 months	0.4	0.5	0.2	0.040
6	14 months	0.4	0.5	0.8	0.160
7	12 months	0.6	0.1	0.2	0.012
8	13 months	0.6	0.1	0.8	0.048
9	13 months	0.6	0.4	0.2	0.048
10	14 months	0.6	0.4	0.8	0.192
11	14 months	0.6	0.5	0.2	0.060
12	15 months	0.6	0.5	0.8	0.240

g. While it looks like we have twelve different scenarios, for purposes of estimating project completion, we really only have five: the project will take 11 months, 12 months, 13 months, 14 months, or 15 months.

- i. There are four ways for it to complete the project in 13 months: scenario #4 OR scenario #5 OR scenario #8 OR scenario #9.
- ii. Since it can't take 1 and 2 months to do the same demolition, we can say these scenarios are mutually exclusive. Thus we can add probabilities:

Total Time	Individual Probabilities				Overall Probability
11 months	0.008				0.008
12 months	0.032	0.032	0.012		0.076
13 months	0.128	0.040	0.048	0.048	0.264
14 months	0.160	0.192	0.060		0.412
15 months	0.240				0.240

iii. Note that while there are four ways to get to 13 months, 14 months is the most likely time-to-completion.