

LECTURE 08: LAW OF LARGE NUMBERS

I. Law of Large Numbers

a. One of the basic rules of statistics is the *law of large numbers*, or as the number of observations increases, the empirical probability will approach the theoretical probability.

b. Example: Coin flipping

i. The theoretical probability of getting “heads” on a coin flip is 0.50.

ii. If you flip a coin once, you’ll get either heads or tails. That means the empirical probability of getting “heads” is either 1.00 or 0.00. That’s way off!

iii. Let’s flip it twice. Here are the possible results:

Result	Chance of Heads
HH	1.00
HT	0.50

Result	Chance of Heads
TH	0.50
TT	0.00

iv. Now you have a 50% chance of getting the theoretical result and a 50% of getting an extreme result.

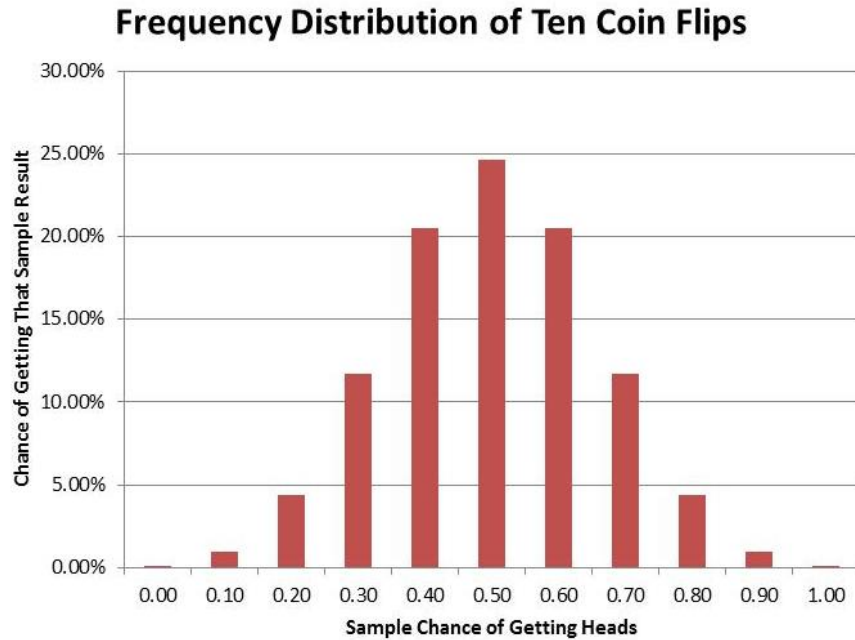
v. Let’s flip it four total times. Here are the possible results:

Result	Chance of Heads
HHHH	1.00
HHHT	0.75
HHTH	0.75
HTHH	0.75
THHH	0.75
HHTT	0.50
HTHT	0.50
THHT	0.50

Result	Chance of Heads
HTTH	0.50
THTH	0.50
TTHH	0.50
TTTH	0.25
TTHT	0.25
THTT	0.25
HTTT	0.25
TTTT	0.00

vi. You may only have a 37.5% chance of getting the theoretical result, but you have only a 12.5% chance of getting one of the extreme results. With the mid-range results each at 25%, the theoretical result is the most likely result to get.

vii. And if you flipped the coin ten times...



II. Gambler's Fallacy

- a. It's tempting to be fooled by the law of large numbers. If black comes up ten times in a row on a roulette wheel, people think that red must be "due." The thinking is that it must be more likely to come up in order to balance out the previous streak. Otherwise, how could we say increasing the sample size brings the sample mean closer to the theoretical mean?
 - i. This is called the *gambler's fallacy*—believing **under-**represented results will be more likely to occur in future independent trials.
- b. But look at our bar graph of ten coin flips: we get ten heads (or ten tails) 0.10% of the time. In other words, it's possible that a streak can continue.
- c. The law of large numbers doesn't render independent trials dependent. The roulette wheel has the same chance of getting black if black came up ten times in a row or red came up ten times in a row.
- d. On August 18, 1913, black came up twenty-six times in a row at the Monte Carlo casino. People bet (and lost) millions on the idea that red "was due" for a streak. But any particular sequence of red and black is just as likely as all black.
- e. Note the gambler's fallacy does not apply to dependent events, such as card-counting.

III. Hot Hand Fallacy

- a. People sometimes succumb to the opposite of the gambler's fallacy, called the *hot hand fallacy*—believing **over**-represented results (particularly successes) will be more likely to occur in future independent trials.
- b. Success now does not mean success later. Just because a basketball player made three shots in a row does not mean they are suddenly more likely to make a fourth shot.
 - i. This fallacy is a bit harder to detect when it comes up in games based at least partly on skill.
- c. Most (but not all) of the evidence analyzing basketball players' success at shooting suggests the result for any given shot (for a particular player) is random.
 - i. These studies tend to focus on free-throws, where you can remove complexities like where the shot was taken from or what the other team is doing.
- d. If you're doing well at craps, that doesn't mean your next roll of the dice will be successful.
- e. If the slot machine you're using pays out, that doesn't mean it will continue to pay out.
- f. Just because a stock is doing well now doesn't mean it will continue to do well.
 - i. The day-to-day and hour-to-hour movement of a particular stock is essentially random. At any particular price, it has an equal chance of going up and of going down.